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SOURCES OF ERROR IN OBJECTIVE ANALYSIS

by

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The error in objective analysis methods that are based on corrections to a first guess field is considered. An expression that gives a decomposition of the error into three independent components is derived. To test the magnitudes of the contribution of each component a series of computer simulations was conducted. Grid-to-observation point interpolation schemes considered ranged from simple piecewise linear functions to highly accurate spline functions. The observation-to-grid interpolation methods

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considered included most of those in present meteorological use, such as optimum interpolation and successive corrections, as well as proposed schemes such as thin plate splines, and several variations of these schemes. The results include an analysis of cost versus skill; this information is summarized in plots for most combinations. The degradation in performance due to inexact parameter specification in statistical observation-to-grid interpolation schemes is addressed. The efficacy of the mean squared error estimates in this situation is also explored.

1.0 Introduction

The purpose of this investigation was to test the relative importance of various aspects of correcting predicted values on a grid by incorporating information from observed values at scattered data points. Grid and observation configurations were patterned after those routinely available over North America. Although investigations were limited to the univariate objective analysis methods, I believe the results are indicative of those that would be achieved in the more general case.

Previous investigations on the error contribution of various steps in the objective analysis process are limited. Koehler (1979) separately studied the errors of a number of grid-to-observation and observation-to-grid interpolation (approximation) routines. He noted that although little attention is typically paid to the grid-to-observation interpolation process significant errors may be caused by this phase of objective analysis. While this may be a surprise since these errors are usually small compared to the first-guess errors at the grid points, my results further demonstrated that the contribution to overall error made by the grid-to-observation interpolation process should not be ignored. This investigation complements recent work by Seaman (1983) regarding the accuracy of statistical and successive correction schemes. His work provides expected mean squared error estimates for these schemes. His work is very thorough in that it provides estimates of the analysis error as the parameters of the first-guess error are varied while holding the assumed values constant, and vice-versa.

In Section 2 I derive a generalized expression for the overall error in objective analysis which leads to several observations. In Section 3 I describe the simulation method and the various options which can be easily handled. In Section 4 I present the results of the simulations and discuss their implications with regard to the observations made in Section 2.

2.0 The Form of the Error Term in Objective Analysis

My setting for study of the objective analysis process assumes the following:

(i) The true field (function) to be analyzed is H.

(ii) H is known imperfectly at grid points through a "first-guess" which is in error by an amount to be denoted by g. The error is a normally distributed stationary random function which has a certain spatial correlation and standard deviation.

(iii) H is imperfectly measured at observation points yielding values with errors o. These errors are independent and normally distributed with certain standard deviation.

The nature of the errors makes it only possible to evaluate g at grid points, and o at observation points, although it is sometimes convenient to think of them as functions rather than as sets of errors. The objective analysis process consists of interpolation of the first-guess values from the grid to the observation points (by a linear operator designated M) followed by interpolation of the difference between the observed and first-guess values back to the grid point (by a linear operator designated L) as a correction to the first-guess values. Denote the error in the entire process by E, then the final approximation is

$$H + E = H + g + L(H + o - M(H + g)) \ .$$

Let $m(H)$ represent the error in the approximation of H by $M(H)$, then $M(H) = H - m(H)$. Rearranging and simplifying the above, leads to

$$\begin{aligned} E &= g + L(H + o - M(H) - M(g)) \\ &= g + L(H + o - H + m(H) - M(g)) \\ &= g + L(o + m(H)) - LM(g) \ , \end{aligned}$$

and finally,

$$E = L(o) + Lm(H) + (g - LM(g)) \ . \tag{1}$$

Thus the error is made up of three parts. The term $L(o)$ is dependent on the 'function' o, which describes instrumentation error and is typically not controlable. It is obviously advantageous to have o small. Since the values of o are assumed independent and random it is desirable for L to be a smoothing operator. The second part, $Lm(H)$ is within our control and the grid-to-observation point interpolations error should be made small. If it is, then interpolation of the error back to the grid points by L is also small, assuming this smoothing operator is typical and does not magnify the error. The third part $(g - LM(g))$ is the error in interpolation of the first-guess error at the grid points to the observation locations by M, then back to the grid points by L. While it is possible that a certain symbiosis between parts could occur, the goal is certainly for each interpolation process to have small errors. Ideally the operator L should be a left inverse of the operator M, although this is almost certainly impossible.

Partitioning the error in this way shows, for example, that

using a better interpolation process from the grid to the observation points should decrease the overall analysis error. In certain realizations, of course, the errors may tend to cancel. Since the three terms represent uncorrelated errors, the total error variance over many realizations will tend to be the sum of the individual variances. Thus, decreasing any one will lead to statistically smaller error variances.

3.0 The Computer Simulation Methods

In order to simulate the behavior of the overall error under various interpolation processes and first-guess error assumptions, a modular computer program was written to give several options for the different processes. This made it possible to test a large number of combinations of methods and assumptions.

In general terms, the process simulated consists of the following steps:

(i) An underlying mathematically defined function describing the field to be analyzed is evaluated on a grid of points.

(ii) "First-guess" error is generated from normal random deviates with a pre-specified standard deviation and spatial correlation.

(iii) "Observed values" are generated by evaluating the field to be analyzed at the observation points, and adding normally distributed uncorrelated random deviates to these values.

(iv) The first-guess values at the observation points are obtained by one of several interpolation schemes.

(v) Based on the difference between first-guess and observed values at the observation locations, "corrected" values at the grid points are obtained. I will refer to the corrected values as the analysis values.

Most of the simulations were done with two different grids and observation point sets. One was based on a 2.5° grid covering 112.5° W to 82.5° W and 30° N to 50° N, with 117 = 13x9 grid points and 36 observation points within the grid, as shown in Figure 1. The other was based on a 5° grid covering 125° W to 75° W, and 25° N to 60° N, with 88 = 11x8 grid points, and 67 observation points within the grid. This grid and the observation locations are shown in Figure 2. All the simulations used were univariate analysis methods on a two-dimensional field. This simplification was necessary for two reasons. The first reason is that the generation of error with a specified spatial correlation required factorization of the correlation matrix into the product of a lower triangular matrix and its transpose. The correlation matrix is of order equal to the number of grid points, and it is not particularly well conditioned. Incorporation of multiple levels, a large grid, or correlated multiple variables was therefore not possible. The other reason is that statistical results required that numerous realizations be simulated, thereby limiting the time available to do the computations.

The underlying mathematically defined field can be any specified function. The height field test function used is the one given by Koehler (1979) and also described in Wahba and Wendelberger (1980). The input parameters, θ_o (the location of

the longitudinal wave), $\Delta\theta$, (amount part of the field is skewed logitudinally), and $\bar{p}$ (the pressure for the height field) are easily varied. The experiments simulated the 500 mb height field, using fixed or randomly varying θ_o and $\Delta\theta$. A typical field of height contours generated by this function is shown in Figure 3. First-guess errors had a nominal standard deviation, r_g, of 30 m. The spatial correlation function was modeled using $\exp((-d/c_d))^2$, where d is distance (on the degree grid), and c_d is a correlation distance, specified as 10^o. I have used degree measure for distance rather than true distance, to maintain a rectangular grid of first-guess points. This resulted in a distortion of the distance varying with location. The observation errors had a nominal standard deviation, r_o, of 10 m. The observation locations approximately correspond to the North American radiosonde network within the grids being used. They are shown, along with the grids, in Figures 1 and 2.

The output consisted of mean, root-mean-square, and maximum errors over each data set (first-guess at grid points, first-guess at observation locations, observation at observation locations, and analysis values at grid points) for each realization. The first and third of these mainly served as a check on the psuedo-random number generator (IMSL subroutines GGNSM and GGNML). The output also gave summaries of the same errors over all realizations as well as the mean and standard deviation of the root-mean-square errors over the realizations. Interpolation processes are sometimes ill-behaved around boundaries. Since in the global problem this can be avoided, the effects were minimized here by tabulating error only over the interior grid

points. Thus the results are over 77 grid points on the 2.5° grid and 54 grid points on the 5° grid. The options simulated for each step are described below.

a. Grid-to-observation point interpolation

First-guess values at the observation points are obtained by interpolation from the first-guess grid values. I compared four schemes. Others could be easily included, however my results indicate it will probably not be fruitful to do so. The methods I have used are:

(i) Piecewise bilinear interpolation. As with any piecewise defined method, one must first determine the rectangle in which the evaluation point lies. Then, the evaluation is most easily seen as translation to the square $[0,1]^2$, followed by 3 one dimensional interpolations. This requires 8 operations, where an operation is defined as a multiplication or division followed by an addition or subtraction. Practically, the evaluation can be accomplished in 5 operations (and a couple of extra additions/subtractions). In my cost analysis I have used 8 operations; this cost is very low compared to that of other necessary calculations.

(ii) Bicubic spline interpolation. I used the IMSL subroutines IBCCCU and IBCEVL. Preprocessing for the spline coefficients on a $N_{\theta} x N_{\phi}$ grid requires $12N_{\theta} x N_{\phi}+27N_{\phi}+51N_{\theta}-10$ operations. Evaluation requires 2 operations to translate to $[0,1]^2$ and 5 cubic interpolations at 9 operations each for a total of 47 operations. The preprocessing operations involve solution of tridiagonal systems of equations which are amenable to vectoriza-

tion for pipeline computers.

(iii) Piecewise bicubic interpolation. My implementation of this scheme used 2 operations for a translation to $[0,3]^2$ followed by 5 cubic interpolations, each costing 5 operations. In addition, a difference table was formed at a cost of several subtractions.

(iv) Bessel bicubic interpolation. My implementation of this scheme used 2 operations for a translation to $[0,3]^2$ followed by 5 cubic interpolations, each costing 5 operations. Because of default to parabolic interpolation in boundary regions, there were some additional tests. There were also a few subtractions to form the difference table.

b. Observation-to-grid point interpolation

As in operational weather forecasting programs, the differences between first-guess and observed values at the observation points are used to correct the first-guess values on the grid to obtain analysis values on the grid. I have tested twelve schemes for performing this correction. I will give a brief description of each method and refer the reader elsewhere for complete details. The first-guess error at the observation location, $P_k = (\theta_k,\phi_k)$, is denoted by ΔH_k, $k=1,\ldots,N_o$. The number of grid points is $N_\theta N_\phi$. I want to evaluate the approximation at grid points, but will write it in terms of a generic point, $P = (\theta,\phi)$. Recall that the standard deviation of the first-guess errors is r_g, and the spatial covariance function is denoted by $C(P,Q)$.

An operation count has been made for each of the methods. I discuss briefly how various phases of the process contribute, and

summarize the results in Table 1, along with some representative numbers that arise from my simulations. I have described some schemes as local, implying that others are global. In the context of global objective analysis, all the schemes I consider are local; the schemes which are global for my simulation are less local than the ones I refer to as local.

(i) Optimum interpolation (OI). This scheme was introduced to the meteorological literature by Gandin (1963) and has received widespread attention in recent years, e.g. see Bergman (1979) and Lorenc (1981). The method in its proper form requires that the spatial covariance function of the first-guess errors and the standard deviation of the observation error be known. Since these are known for this simulation, I have used their properties. I have implemented the scheme as described in Franke and Gordon (1983), viewing the approximation as a linear combination of the covariance functions associated with the observation points. Thus we have

$$\Delta H(P) = \sum_{k=1}^{N_o} a_k C(P,P_k) ,$$

where $C(P,Q)$ is as noted above. The a_k are determined from the system of equations

$$\left(C(P_i,P_j) + \delta_{ij} r_o^2\right) \begin{pmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_{N_o} \end{pmatrix} = \begin{pmatrix} \Delta H_1 \\ \cdot \\ \cdot \\ \cdot \\ \Delta H_{N_o} \end{pmatrix}$$

where ΔH_i is the difference between the first-guess and observed values at the i^{th} observation point, r_o is the standard deviation of the observation error, and δ_{ij} is the Kronecker delta.

The cost of (OI) consists of a preprocessing phase that

includes the generation and solution of the system of equations, followed by evaluation of the analysis at the grid points. For N_o observations, preprocessing is at a cost of $N_o(N_o+1)/2$ function evaluations to generate the coefficient matrix and $(N_o^3+6N_o^2-N_o)/6$ operations plus N_o square roots to perform Cholesky decomposition and solution of the system of equations for the a_k. Evaluation costs N_o covariance function evaluations and N_o operations to form the linear combination representing the value of the correction at each grid point.

(ii) Local optimum interpolation. In my version of this scheme, nominally only points within the surrounding 10^o square are used; if fewer than 4 observations are available, the square is expanded to 15^o and so on, by 2.5^o increments in each direction until at least 4 observations are available. The costs of the search were not assessed. For each grid value correction, a system of equations must be formed and solved, and the corresponding correction computed. With n observations being used the expressions given for OI above apply with n replacing N_o. This process was performed for each grid point, making the total cost the sum of these costs over all grid points.

(iii) Global Barnes' method. This type of scheme is described by Barnes (1973) and others. My scheme used the known correlation functions as the weights for the first pass. Thus, the approximation is

$$\Delta H_{\cdot,1}(P) = \sum_{k=1}^{N_o} W_k(P)\Delta H_k / \sum_{k=1}^{N_o} W_k(P) ,$$

where $W_k = \exp((-(||P-P_k||/c_d)^2)$, and ΔH_k is as before. For the

second pass the correction has the same form, but ΔH_k is replaced by $\Delta H_{k,1}$, the difference between the corrected first-guess and the observations. The quantity c_d is replaced by $c_d/3^{1/2}$ for the second pass. The total correction at the grid points is then the sum of the two corrections. For each grid point the cost of this method is N_o weight function evaluations per pass and N_o+1 operations per pass. In addition a separate interpolation from the grid points to the observation points is required before the second pass. This type of scheme has been defined and studied in a different context, without a change of weight functions between iterations, by Foley and Nielson (1980).

(iv) Local Barnes' method. The same localization process as used for the local OI scheme (ii) was used here. As for the global version, two passes were used. Hence the cost for an evaluation at a grid point with n neighboring points is the same expression as in the global scheme, but with n replacing N_o. In addition, there was the search cost to determine the nearby observations, which was not assessed. Costs of an interpolation from the grid points to the observation points between passes was included.

(v) Statistical interpolation ($c_d = 14^o$). In practical applications of OI the error correlations and standard deviations cannot be modeled precisely. This has lead to the use of the name "statistical interpolation". Computationally the method is identical to the OI scheme (i). Here the only difference is the substitution of an inexact correlation distance, $c_d = 14$. The algorithm and costs are identical.

(vi) Statistical interpolation ($c_d = 7^o$). Again this is

identical to (i) except that the inexact value substituted for c_d is 7.

(vii) Statistical interpolation (damped cosine correlation function). Once more this scheme is computationally identical to (i) except that the correlation function used is of the form $\exp((-(||P-Q||/c_d)^2)\cos((||P-Q||/c_d)(/2))$. I used the value $c_d = 10$.

(viii) Thin plate splines. This method is described by Wahba and Wendlelberger (1980) and others. The approximating function used by the scheme is

$$H(P) = \sum_{k=1}^{N_o} A_k B(P,P_k) + a\theta + b\phi + c ,$$

where the basis function $B(P,Q) = ||P-Q||^2 \log||P-Q||$. The A_k and a, b, and c, are obtained by solving the system of equations

$$\sum_{j=1}^{N_o} A_k (B(P_i,P_j)+\lambda N_k r_o^2 \; {}_{ij})+a\theta_i+b\phi_i+c = \Delta H_i \; ,i=1,\ldots,N_o$$

$$\sum_{j=1}^{N_o} A_j\theta_j = 0$$

$$\sum_{j=1}^{N_o} A_j\phi_j = 0$$

$$\sum_{j=1}^{N_o} A_j = 0 .$$

In the above, λ is a smoothing parameter. The smoothing parameter was chosen on the basis of a few trials with no attempt to optimize its choice for a particular data set, as can be done. Wendelberger (1981) describes a program that will automatically choose λ (and m as well, see next method), but I have not tested

it yet. This system of equations is symmetric, but not positive definite. I have used standard L-U decomposition routines to solve the system. Methods for symmetric indefinite systems use about half as many operations, however I observed greater numerical stability using the general decomposition process. There are $N_o(N_o+1)/2$ basis function evaluations, and solution of the system of equations requires $(N_o+3)(N_o^2+6N_o+3)/3 + (N_o+3)^2$ operations. Unlike symmetric positive definite systems, solution of these equations requires searching for a pivot and pivoting. Evaluation at each grid point then requires N_o basis function evaluations and N_o+2 operations to form the sum.

(ix) Laplacian smoothing spline (m=3). This scheme is also described by Wahba and Wendelberger (1980), and is one of those available in the program by Wendelberger (1981). The thin plate spline method is a member of this family (with m=2), but also has the "thin plate" interpretation. The reason for inclusion of this method is that the results of Wahba and Wendelberger indicate that pressure height fields are better approximated using values of m = 3 or 4. I will not describe the method fully. It requires evaluation of $N_o(N_o+1)/2$ basis functions and $3N_o$ multiplications to set up the system of N_o+6 equations to be solved. Then N_o+5 operations would be required for evaluation at each grid point, along with the evaluation of N_o basis functions.

(x) Franke/Gordon. This scheme was suggested by Franke and Gordon (1983) as one which is an explicit scheme, similar to Barnes' method, but which when iterated converges to the OI interpolant. Three iterations, with the parameter $= .85||M||$

(in the notation of that report) were performed. The cost in operations is $2N_o(N_o+1)$ plus $3N_o$ for each grid point. The number of weight function evaluations is $2N_o^2$ plus $3N_o$ for each grid point.

(xi) Pseudo-Barnes' method. This method was described in Franke and Gordon (1983) and was at that time mistaken for Barnes' method. It differs in that the error at the second iteration is Barnes' approximation evaluated at the observation point minus the first-guess error, rather than the the corrected first-guess at the grid point interpolated to the observation point minus the observed value. The cost of this algorithm is evaluation of N_o^2 weight functions plus $2N_o$ for each grid point. It requires $N_o(N_o+1)$ operations, plus $2(N_o+1)$ for each grid point.

(xii) Local pseudo-Barnes' method. This is a local version of (xi), using the same "nearby" observation points as (ii) and (iv). A grid point with n nearby observation points requires evaluation of n^2+2n basis functions and n^2+3n+2 operations.

4.0. Results

The simulation program described in the previous section was run for a substantial number of different options. Each run consisted of 100 realizations of a test field each containing associated first-guess and observation errors. Table 2 gives the assumed parameter values for the various cases. Not all combinations of grid-to-observation point and observation-to-grid point interpolation schemes were used in every case. The tables detail the complete results and the entries indicate which combinations were computed. Each combination in a given table (3-14)

corresponds to the same set of realizations, but different tables depend on different realizations.

This investigation was designed to determine the influence of the grid-to-observation point interpolation scheme. This influence is seen by noting changes in error for a particular observation-to-grid point interpolation scheme as the grid-to-observation point interpolation scheme is varied. The rows of Tables 3-14 give this information. The bicubic spline interpolation produced significant improvement over piecewise bilinear interpolation. This verifies the smaller magnitude of the term Lm(H) in the error expression given by (1) for the spline method. For 2.5° grids the errors were no smaller for spline interpolation than for piecewise bicubic or Bessel bicubic interpolation. Evidently the grid spacing was small enough (for the test function used) that the interpolation error was not significant. Spline interpolation did show an improvement over piecewise bicubic and Bessel bicubic interpolation on the 5° grid. Spline interpolation and the cubic interpolation methods showed even greater improvement over piecewise linear interpolation on the 5° grid than on the 2.5° grid. Interestingly the first-guess errors at the observation points had greater rms values for cubic interpolation than they did for linear interpolation. This occurs because linear interpolation inherently has greater smoothing.

Most of the useful information given in Tables 3-14 can be more easily obtained from plots of the salient values. Figures 4-8 give plots of skill vs. cost of the algorithm in thousands of

operations per analysis. Here "skill" is defined to be 1 - rmsa/r_0, where rmsa is the rms error in the analysis values. The skill with respect to bilinear and bicubic interpolation are each indicated, connected with a straight line to delineate the extent between the two. The results for only one of the statistical schemes, (vi), has been plotted since the others were nearly identical. For these purposes I counted an evaluation of a basis, weight, square root, or covariance function as 10 operations. The plots reveal that the statistical schemes, local OI, and thin plate splines all had close to the same accuracy and all were slightly less accurate than OI. The Barnes' schemes, the Franke/Gordon scheme, and Laplacian smoothing splines were least accurate. The poor performance of the Laplacian smoothing splines here, in contrast to the better performance obtained by Wahba and Wendelberger (1980) is probably due to the scheme being applied to the first-guess error function rather than to the underlying true height field. The degradation in the performance of the less than optimal statistical schemes is perhaps less drastic than one might expect. It does appear that it was better to underestimate the correlation distance than to overestimate it.

Figure 9 shows plots of the rms errors in the analysis values as a function of first-guess errors. The improvement in the Barnes' scheme as the first-guess errors decrease was rapid. The scheme gave results nearly as good as OI, the statistical schemes, and thin plate splines. This occurred because the principal problem became smoothing observation errors as the first-guess errors tended to zero. Figure 10 shows plots of the

rms errors in the analysis values as a function of observation errors. As observation errors go to zero the importance of modelling the first-guess error was more important than smoothing. Thus OI, the statistical schemes, and thin plate splines improved the most, while both Barnes' schemes improved little. Figures 11-13 show the rms errors in the analysis values when incorrect variances were specified for the interpolation routines. Methods not using these values were naturally unaffected so that changes in the rms errors in the analysis values for these methods only reflect the variability of the (different) realizations used in the various cases. The plots show that the use of incorrect values for the first-guess and observation error variances did not drastically affect the accuracy of the statistical methods. The interested reader is referred to Seaman (1983) for more extensive tests of the effects of incorrect parameter specification on the performance of statistical interpolation methods.

One of the attractive features of the statistical schemes is that they afford a calculation for the estimated mean squared error. These estimates do not depend on any particular realization, so they were not incorporated into the process. However, I did compute them as a side calculation for my grids and observation points. The results of these calculations are tabulated for the 2.5° grid, along with the empirical rms errors obtained during the simulations. Table 15 shows that the estimates given by OI were quite good; the estimated and empirical errors varied only a few percent. They also were accurate for local OI, as

they should be. On the other hand, the slight degradation in performance of statistical methods when incorrect correlations or variances were specified did not carry over to the error estimates. In fact the schemes that have their performance degraded the most (in this case, using too long a correlation distance) showed a decrease in the estimated error variance. Conversely, shortening the correlation distance in the statistical method increased the error estimate as well as the empirical error obtained, although the empirical error is underestimated. This indicates that one must not put too much faith in the error estimates when the actual covariance structure is not known, as in practice. It appears one could obtain just about any error estimate wished simply by specifying unrealistic parameters for the covariance structure.

The principal results of this study were as follows. The decomposition of the error into independent components in (1) identified possible ways to decrease the analysis error. This lead to the results showing the contribution of the grid-to-observation interpolation process, the necessity of smoothing in the observation-to-grid interpolation process, along with accuracy. The simulations provided confirmation of the above and yielded information concerning the sensitivity of statistical interpolation schemes to inexact parameter specification. The error estimates provided by statistical schemes were shown to be sensitive to inexact parameter specification.

5.0 Acknowlegdements

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Method	1: #Operations	2: #Basis	3: Other	Example # (1,2,3) 2.5° grid, 36 obs 5° grid, 67 obs
Opt Interp	$(N_o^3+6N_o^2-N_o)/6$ $+N_oN_\theta N_\phi$	$N_o(N_o+1)/2+N_oN_\theta N_\phi$	N_o sqrts	13278, 4878, 36 60501, 8174, 67
Local OI	$\Sigma((n^3+6n^2-n)/6+n)$	$\Sigma(n(n+1)/2+n)$	search, Σn sqrts	9655, 3166, 683 7333, 2400, 516
Barnes'	$2N_\theta N_\phi(N_o+1)$	$2N_oN_\theta N_\phi$	grid-to-obs	8658, 8424, 972 11958, 11792, 1809
Local Barnes'	$\Sigma(2n+2)$	$\Sigma 2n$	search, grid-to-obs	1600, 1366, 972 1208, 1032, 1809
Thin Pl Spl	$(N_o+3)((N_o+3)^2-1)/3$ $+(N_o+3)^2+(N_o+2)N_\theta N_\phi$	$N_o(N_o+1)/2+N_oN_\theta N_\phi$	--	25725, 4878, - 125281, 8174, -
Lapl Sm Spl	$(N_o+6)((N_o+6)^2-1)/3$ $+(N_o+6)^2+(N_o+5)N_\theta N_\phi$	$N_o(N_o+1)/2+N_oN_\theta N_\phi$	--	31585, 4878, - 141690, 8174, -
Frnke/Grdn	$3N_\theta N_\phi(N_o+1)+2N_o(N_o+1)$	$3N_oN_\theta N_\phi$	--	15651, 15288, - 27064, 26665, -
Pseudo-Barnes'	$2N_\theta N_\phi(N_o+1)+N_o(N_o+1)$	$2N_oN_\theta N_\phi$	--	9990, 9720, - 16524, 16281, -
Local P-Barnes'	$\Sigma(n^2+3n+2)$	$\Sigma(n^2+2n)$	search	6419, 5649, - 4076, 4284, -

Table 1

Table#	r_g	r_o	θ_o	θ	grid	notes[c]
3	30	10	100	0	13x9,2.5°	
4	30	10	100	13.775	13x9,2.5°	
5	20	10	100	0	13x9,2.5°	
6	30	5	100	0	13x9,2.5°	
7	30	10	random[a]	random[b]	13x9,2.5°	
8	30	10	random[a]	random[b]	13x9,2.5°	$(r_o)_i = 30$
9	20	10	random[a]	random[b]	13x9,2.5°	$(r_g)_i = 20$
10	30	10	random[a]	random[b]	13x9,2.5°	$(r_o)_i = 5$
11	30	5	random[a]	random[b]	13x9,2.5°	$(r_o)_i = 10$
12	5	10	random[a]	random[b]	13x9,2.5°	
13	30	0	random[a]	random[b]	13x9,2.5°	
14	30	10	random[a]	random[b]	11x8,5°	

Table 2

a θ_o uniformly distributed in (-82.5°,112.5°)
b $\Delta\theta$ uniformly distributed in (-15°,15°)
c The statistical interpolation routines were given incorrect variances, as indicated

P = 500, Theta= 100, Delth = 0 Entries: RMSE Analysis
rg = 30, ro = 10 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 observation points

Grid-to-obs: / Obs-to-grid	PW linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	6.64 6.53(1.18)	6.09 5.97(1.20)	6.09 5.98(1.19)	6.09 5.98(1.19)
Local OI (Cd= 10)	7.09 6.99(1.19)	6.53 6.42(1.19)	6.54 6.44(1.19)	6.55 6.44(1.18)
Barnes' 2-Pass	9.27 9.08(1.03)	8.87 8.68(1.82)	8.87 8.68(1.82)	8.88 8.69(1.82)
Barnes' (Local)	8.42 8.27(1.57)	7.95 7.79(1.56)	7.96 7.80(1.56)	7.96 7.81(1.56)
Stat Interp (Cd = 14)	7.28 7.23(1.22)	6.78 6.66(1.27)	6.79 6.67(1.26)	6.79 6.68(1.26)
Stat Interp (Cd = 7)	7.34 7.23(1.25)	6.87 6.75(1.26)	6.87 6.76(1.25)	6.87 6.76(1.25)
Stat Interp (Dmpd Cos)	7.37 7.26(1.28)	6.91 6.79(1.28)	6.91 6.79(1.28)	6.91 6.79(1.28)
Thin Pl Spl (m = 2)	7.12 7.00(1.30)	6.59 6.45(1.33)	6.60 6.46(1.32)	6.60 6.47(1.32)
Lapl Sm Spl (m = 3)	10.54 10.40(1.73)	10.25 10.10(1.73)	10.25 10.10(1.73)	10.25 10.11(1.73)
Frnke/Grdn (3 Pass)	12.02 11.72(2.65)	11.75 11.45(2.65)	11.76 11.45(2.65)	11.76 11.45(2.65)
PseudoBarnes' (2 Pass)	9.28 9.10(1.83)	8.87 8.68(1.82)	8.87 8.68(1.82)	8.88 8.69(1.82)
PseudoBarnes' (Local)	8.20 8.06(1.51)	7.70 7.55(1.50)	7.71 7.57(1.50)	7.72 7.57(1.50)

TABLE 3

P = 500, Theta = 100, Delth = 13.775 Entries: RMSE Analysis
rg = 30, ro = 10 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	6.88 6.72(1.49)	6.19 6.05(1.34)	6.20 6.06(1.34)	6.21 6.06(1.34)
Local OI (Cd = 10)	7.40 7.24(1.51)	6.76 6.62(1.37)	6.77 6.63(1.37)	
Barnes' 2-Pass	9.73 9.53(1.96)	9.28 9.09(1.88)	9.29 9.10(1.88)	
Barnes' (Local)	8.29 8.16(1.48)	7.79 7.67(1.36)	7.77 7.66(1.36)	
Stat Interp (Cd = 14)	7.71 7.56(1.50)	7.08 6.95(1.35)	7.09 6.96(1.35)	
Stat Interp (Cd = 7)	7.54 7.39(1.50)	6.96 6.82(1.36)	6.96 6.83(1.36)	
Stat Interp (Dmpd Cos)	7.37 7.26(1.28)	6.91 6.79(1.28)	6.91 6.79(1.28)	6.91 6.79(1.28)
Thin Pl Spl (m = 2)	7.45 7.29(1.51)	6.80 6.66(1.37)	6.81 6.67(1.37)	
Lapl Sm Spl (m = 3)				
Frnke/Grdn (3 Pass)				
PseudoBarnes' (2 Pass)	9.75 9.55(1.97)	9.28 9.09(1.88)		
PseudoBarnes' (Local)				

TABLE 4

P = 500, Theta = 100, Delth = 0 Entries: RMSE Analysis
rg = 20, rc = 10 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	6.23 6.10(1.28)	5.75 5.62(1.22)	5.76 5.63(1.22)	
Local OI (Cd = 10)	6.54 6.41(1.28)	6.10 5.97(1.22)	6.10 5.98(1.22)	
Barnes' 2-Pass	7.18 7.03(1.47)	6.85 6.71(1.47)	6.86 6.71(1.41)	
Barnes' (Local)	7.19 7.08(1.22)	6.77 6.68(1.11)	6.76 6.67(1.10)	
Stat Interp (Cd = 14)	6.70 6.58(1.27)	6.30 6.19(1.19)	6.31 6.19(1.19)	
Stat Interp (Cd = 7)	6.71 6.58(1.30)	6.24 6.12(1.25)	6.25 6.12(1.24)	
Stat Interp (Dmpd Cos)	6.78 6.65(1.33)	6.31 6.18(1.28)	6.32 6.18(1.28)	
Thin Pl Spl (m = 2)	6.66 6.52(1.34)	6.20 6.06(1.29)	6.20 6.07(1.29)	
Lapl Sm Spl (m = 3)	11.05 10.84(2.16)	10.71 10.50(2.12)	10.71 10.50(2.12)	
Frnke/Grdn (3 Pass)	8.72 8.54(1.78)	8.56 8.39(1.70)	8.56 8.39(1.70)	
PseudoBarnes' (2 Pass)	7.19 7.03(1.47)	6.85 6.85(1.41)	6.86 6.71(1.41)	
PseudoBarnes' (Local)	6.78 6.66(1.28)	6.37 6.25(1.24)	6.38 6.26(1.23)	

TABLE 5

B = 500, Theta = 100, Delth = 0 Entries: RMSE Analysis
rg = 30, ro = 5 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	4.57 4.50 (0.83)	3.76 3.70 (0.69)	3.77 3.70 (0.69)	
Local OI (Cd = 10)	5.03 4.95 (0.91)	4.26 4.19 (0.77)	4.27 4.20 (0.77)	
Barnes' 2-Pass	8.88 8.69 (1.85)	8.48 8.29 (1.76)	8.49 8.30 (1.77)	
Barnes' (Local)	6.39 6.30 (1.10)	5.90 5.79 (1.11)	5.89 5.78 (1.11)	
Stat Interp (Cd = 14)	5.26 5.17 (0.92)	4.57 4.49 (0.83)	4.57 4.50 (0.83)	
Stat Interp (Cd = 7)	5.02 4.95 (0.82)	4.28 4.21 (0.74)	4.28 4.21 (0.74)	
Stat Interp (Dmpd Cos)	4.96 4.89 (0.83)	4.22 4.16 (0.75)	4.22 4.16 (0.75)	
Thin Pl Spl (m = 2)	4.92 4.85 (0.84)	4.15 4.09 (0.72)	4.16 4.10 (0.72)	
Lapl Sm Spl (m = 3)	6.08 5.98 (1.05)	5.43 5.33 (1.06)	5.43 5.33 (1.06)	
Frnke/Grdn (3 Pass)	11.82 11.55 (2.55)	11.61 11.34 (2.51)	11.61 11.34 (2.50)	
PseudoBarnes' (2 Pass)	8.89 8.70 (1.85)	8.48 8.29 (1.76)	8.49 8.30 (1.77)	
PseudoBarnes' (Local)	7.30 7.15 (1.47)	6.75 6.60 (1.40)	6.76 6.62 (1.41)	

TABLE 6

P = 500, Theta = RANDOM, Delth = RANDOM, Entries: RMSE Analysis
rg = 30, ro = 10 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: / Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	7.00 6.85(1.44)		6.40 6.27(1.30)	
Local OI (Cd = 10)	7.48 7.33(1.49)		6.90 6.90(1.36)	
Barnes' 2-Pass	9.92 9.70(2.07)		9.52 9.33(1.89)	
Barnes' (Local)	8.34 8.21(1.46)		7.89 7.76(1.39)	
Stat Interp (Cd = 14)	7.92 7.77(1.56)		7.41 7.27(1.43)	
Stat Interp (Cd = 7)	7.56 7.41(1.51)		7.01 6.88(1.36)	
Stat Interp (Dmpd Cos)	7.58 7.43(1.53)		7.04 6.90(1.38)	
Thin Pl Spl (m = 2)	7.63 7.47(1.58)		7.06 6.92(1.42)	
Lapl Sm Spl (m = 3)				
Frnke/Grdn (3 Pass)				
PseudoBarnes' (2 Pass)				
PseudoBarnes' (Local)				

TABLE 7

P = 500, Theta = RANDOM, Delth = RANDOM, Entries: RMSE Analysis
rg = 30, rc = 10, rg(lie) = 20 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: / Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	7.14 6.97(1.52)		6.47 6.32(1.35)	
Local OI (Cd = 10)	7.56 7.39(1.62)		6.92 6.77(1.41)	
Barnes' 2-Pass	9.64 9.42(2.04)		9.21 9.01(1.90)	
Barnes' (Local)	8.30 7.04(1.43)		7.73 7.62(1.33)	
Stat Interp (Cd = 14)				
Stat Interp (Cd = 7)				
Stat Interp (Dmpd Ccs)				
Thin Pl Spl (m = 2)	7.19 7.04(1.45)		6.52 6.39(1.26)	
Lapl Sm Spl (m = 3)				
Frnke/Grdn (3 Pass)				
PseudoBarnes' (2 Pass)				
PseudoBarnes' (Local)				

TABLE 8

P = 500, Theta = RANDOM, Delth = RANDOM, Entries: RMSE Analysis
rg = 20, rc = 10, rg(lie) = 30 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: / Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	6.32 6.17(1.36)		5.72 5.60(1.21)	
Local OI (Cd = 10)	6.60 6.47(1.33)		6.05 5.94(1.16)	
Barnes' 2-Pass	7.10 6.95(1.45)		6.69 6.55(1.39)	
Barnes' (Local)	7.09 6.98(1.22)		6.54 6.45(1.08)	
Stat Interp (Cd = 14)				
Stat Interp (Cd = 7)				
Stat Interp (Drpd Ccs)				
Thin Pl Spl (m = 2)	6.31 6.17(1.30)		5.75 5.63(1.16)	
Lapl Sm Spl (m = 3)				
Frnke/Grdn (3 Pass)				
PseudoBarnes' (2 Pass)				
PseudoBarnes' (Local)				

TABLE 9

P = 500, Theta = RANDOM, Delth = RANDOM, Entries: RMSE Analysis
rg = 30, rc = 10, ro(lie) = 5 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: / Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	7.51 7.37(1.43)		6.88 6.74(1.38)	
Local OI (Cd = 10)	7.96 7.85(1.36)		7.37 7.26(1.27)	
Barnes' 2-Pass	9.81 9.60(2.01)		9.28 9.06(1.99)	
Barnes' (Local)	8.47 8.36(1.32)		7.91 7.80(1.31)	
Stat Interp (Cd = 14)				
Stat Interp (Cd = 7)				
Stat Interp (Dmpd Ccs)				
Thin Pl Spl (m = 2)	7.65 7.51(1.43)		6.94 6.80(1.39)	
Lapl Sm Spl (m = 3)				
Frnke/Grdn (3 Pass)				
PseudoBarnes' (2 Pass)				
PseudoBarnes' (Local)				

TABLE 10

P = 500, Theta = RANDOM, Delth = RANDOM, Entries: RMSE Analysis
rg = 30, ro = 5, ro(lie) = 10 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: / Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	5.18 5.10(0.91)		4.19 4.13(0.71)	
Local OI (Cd = 10)	5.66 5.56(1.04)		4.76 4.69(0.81)	
Barnes' 2-Pass	9.09 8.93(1.69)		8.62 8.47(1.61)	
Barnes' (Local)	6.71 6.63(1.07)		6.06 5.97(1.04)	
Stat Interp (Cd = 14)				
Stat Interp (Cd = 7)				
Stat Interp (Drpd Ccs)				
Thin Pl Spl (m = 2)	6.13 6.02(1.15)		5.34 5.25(0.95)	
Lapl Sm Spl (m = 3)				
Frnke/Grdn (3 Pass)				
PseudoBarnes' (2 Pass)				
PseudoBarnes' (Local)				

TABLE 11

P = 50C, Theta = RANDCM, Delth = RANDOM, Entries: RMSE Analysis
rg = 5, rc = 10 Mean RMSE(StDev)
Number cf realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-cbs: / Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 1C)	3.58 3.44(1.01)		3.32 3.18(0.94)	
Local OI (Cd = 10)	3.77 3.63(1.02)		3.53 3.40(0.9769	
Barnes' 2-Pass	4.44 4.32(1.01)		3.98 3.87(0.90)	
Barnes' (Lccal)	6.26 6.15(1.16)		5.73 5.64(1.00)	
Stat Interp (Cd = 14)	3.61 3.46(1.03)		3.41 3.27(0.96)	
Stat Interp (Cd = 7)	3.65 3.50(1.03)		3.39 3.25(0.97)	
Stat Interp (Dmpd Ccs)	3.78 3.62(1.07)		3.51 3.36(1.02	
Thin Pl Spl (m = 2)	4.00 3.84(1.13)		3.85 3.70(1.07)	
Lapl Sm Spl (m = 3)				
Frnke/Grdn (3 Pass)				
PseudoBarnes' (2 Pass)				
PseudoBarnes' (Lccal)				

TABLE 12

E = 500, Theta = RANDOM, Delth = RANDOM, Entries: RMSE Analysis
rg = 30, rc = 0 Mean RMSE(StDev)
Number of realizations = 100
13X9 grid of 2.5 degrees, 36 Observation points

Grid-to-obs: / Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	6.06 5.84(1.62)		0.74 0.69(0.27)	
Local OI (Cd = 10)	3.86 3.79(0.76)		2.40 2.2.30(0.69	
Barnes' 2-Pass	9.38 9.16(2.05)		8.96 8.74(1.98)	
Barnes' (Local)	6.27 6.15(1.21)		5.63 5.49(1.23)	
Stat Interp (Cd = 14)	7.62 7.10(2.74)		1.23 1.16(0.40)	
Stat Interp (Cd = 7)	3.70 3.64(0.68)		1.29 1.20(0.48)	
Stat Interp (Dmpd Cos)	4.48 4.37(0.96)		1.03 0.97(0.36	
Thin Pl Spl (m = 2)	3.74 3.66(0.78)		2.18 2.02(0.83)	
Lapl Sm Spl (m = 3)				
Frnke/Grdn (3 Pass)				
PseudoBarnes' (2 Pass)				
PseudoBarnes' (Local)				

TABLE 13

P = 500, Theta = 100, Delth = 0   Entries: RMSE Analysis
rg = 30, ro = 10   Mean RMSE(StDev)
Number of realizations = 100
11 BY 8 grid of 5 degrees, 67 Observation points

Grid-to-obs: / Obs-to-grid	PW Linear	Bicub Spl	PW Bicub	Bsl Bicub
Opt Interp (Cd = 10)	12.84 12.74(1.62)	7.62 7.53(1.19)	7.93 7.84(1.20)	8.11 8.02(1.19)
Local OI (Cd = 10)	13.33 13.22(1.73)	8.44 8.33(1.33)	8.74 8.63(1.35)	8.92 8.82(1.32)
Barnes' 2-Pass	14.33 14.21(1.85)	10.62 10.49(1.70)	10.82 10.68(1.70)	10.95 10.81(1.71)
Barnes' (Local)	14.00 13.91(1.55)	8.82 8.71(1.36)	9.12 9.79(1.35)	9.40 9.31(1.33)
Stat Interp (Cd = 14)	13.75 13.25(1.57)	8.80 8.70(1.33)	9.02 8.92(1.31)	9.20 9.10(1.32)
Stat Interp (Cd = 7)	13.44 13.35(1.63)	8.31 8.23(1.18)	8.62 8.53(1.21)	8.79 8.70(1.19)
Stat Interp (Dmpd Cos)	13.57 13.47(1.65)	8.47 8.38(1.25)	8.78 8.68(1.27)	8.95 8.86(1.25)
Thin Pl Spl (m = 2)	13.17 13.07(1.59)	8.08 7.99(1.17)	8.36 8.27(1.19)	8.47 8.39(1.19)
Lapl Sm Spl (m = 3)	17.12 17.01(1.86)	11.87 11.76(1.60)	12.08 11.97(1.57)	12.16 12.05(1.58)
Frnke/Grdn (3 Pass)	17.29 17.14(2.29)	15.23 15.04(2.35)	15.31 15.13(2.34)	15.36 15.17(2.37)
PseudoBarnes' (2 Pass)	14.27 14.14(1.87)	10.62 10.49(1.70)	10.82 10.69(1.70)	10.95 10.82(1.71)
PseudoBarnes' (Local)	13.89 13.77(1.82)	9.73 9.60(1.58)	10.00 9.87(1.58)	10.17 10.05(1.57)

TABLE 14

Estimated and (empirical) RMS errors for statistical methods

method	rg: 5 ro: 10	rg: 20 ro: 10	rg: 30 ro: 10	rg: 30 ro: 5	rg: 30 ro: 0
Opt Interp (cd = 10)	3.30 (3.32)	5.62 (5.76)	6.29 (6.40)	3.78 (3.77)	0.65 (0.74)
Local OI (cd = 10)	3.55 (3.53)	5.96 (6.10)	6.80 (6.90)	4.39 (4.27)	2.21 (2.40)
Stat Interp (cd = 14)	2.90 (3.41)	4.60 (6.31)	5.07 (7.41)	2.95 (4.57)	0.11 (1.23)
Stat Interp (cd = 7)	3.76 (3.39)	7.14 (6.25)	8.26 (7.01)	5.46 (4.28)	2.90 (1.29)
Stat Interp (Dmpd Cos)	3.82 (3.51)	7.00 (6.32)	7.94 (7.04)	5.03 (4.22)	2.06 (1.03)

TABLE 15

2.5 DEGREE GRID AND OBSERVATION LOCATIONS

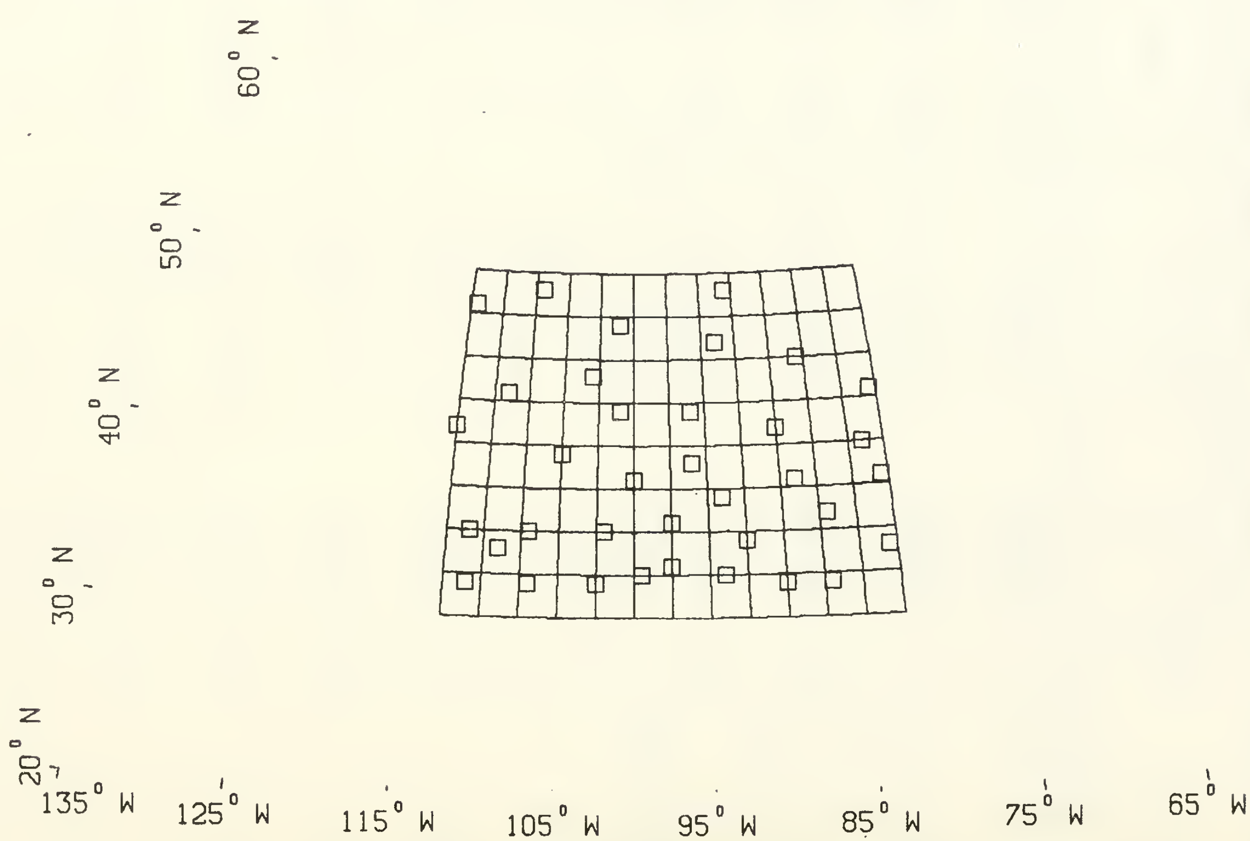


Figure 1

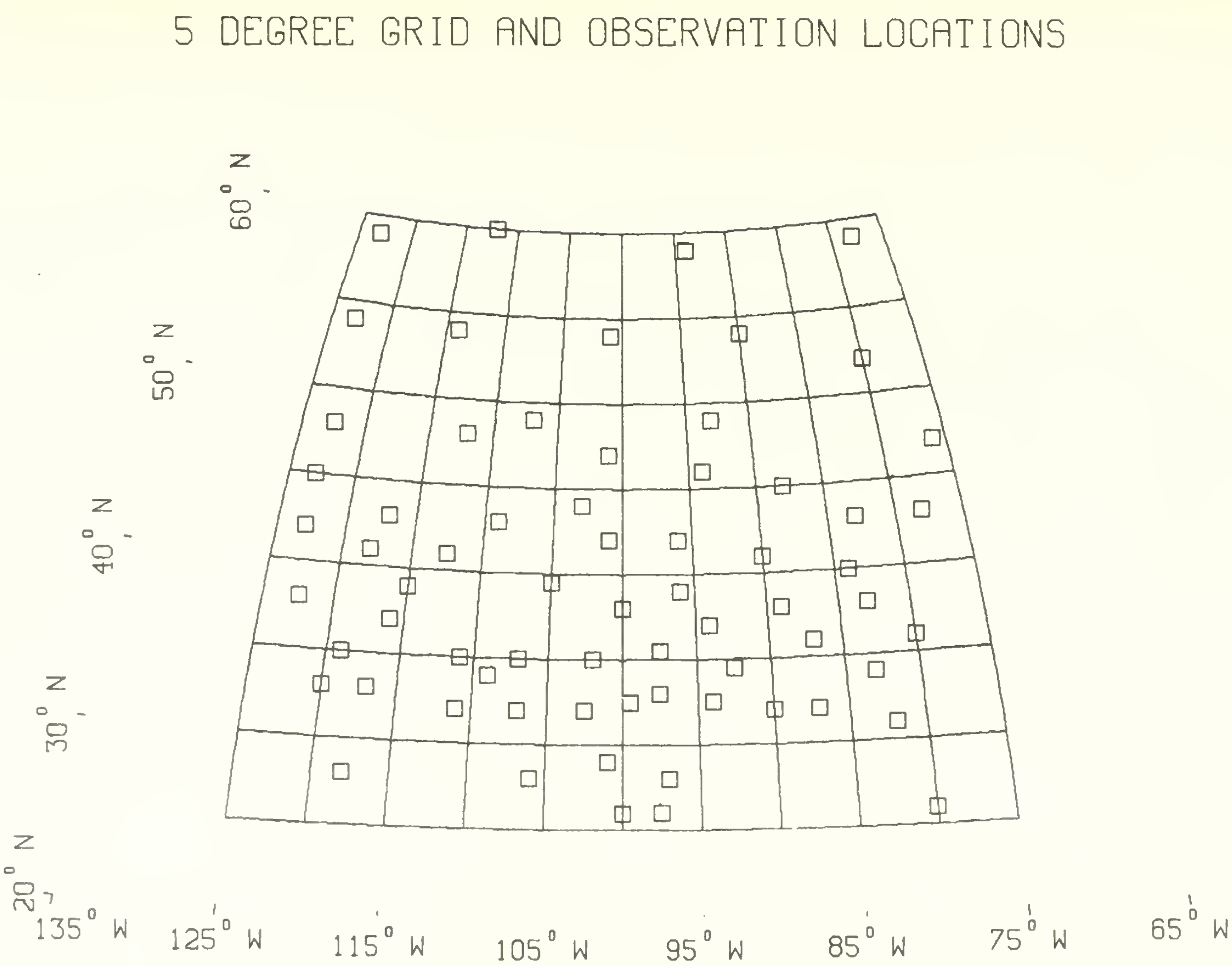

5 DEGREE GRID AND OBSERVATION LOCATIONS
60° N
50° N
40° N
30° N
20° N
135° W
125° W
115° W
105° W
95° W
85° W
75° W
65° W

Figure 2

500 MB CONTOURS, THETA = 100, DELTH = 6

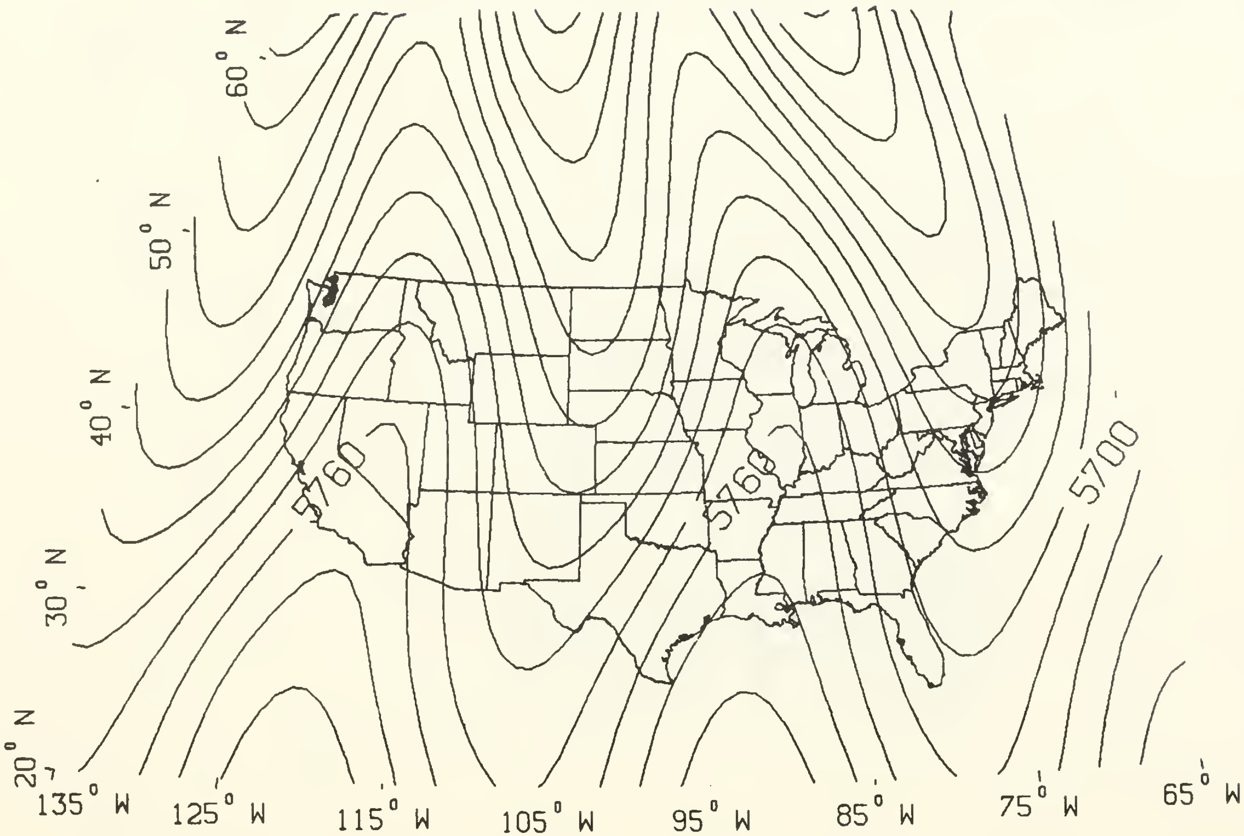


Figure 3

T3: RG = 30, RO = 10, 13X9 2.5 DEGREE GRID

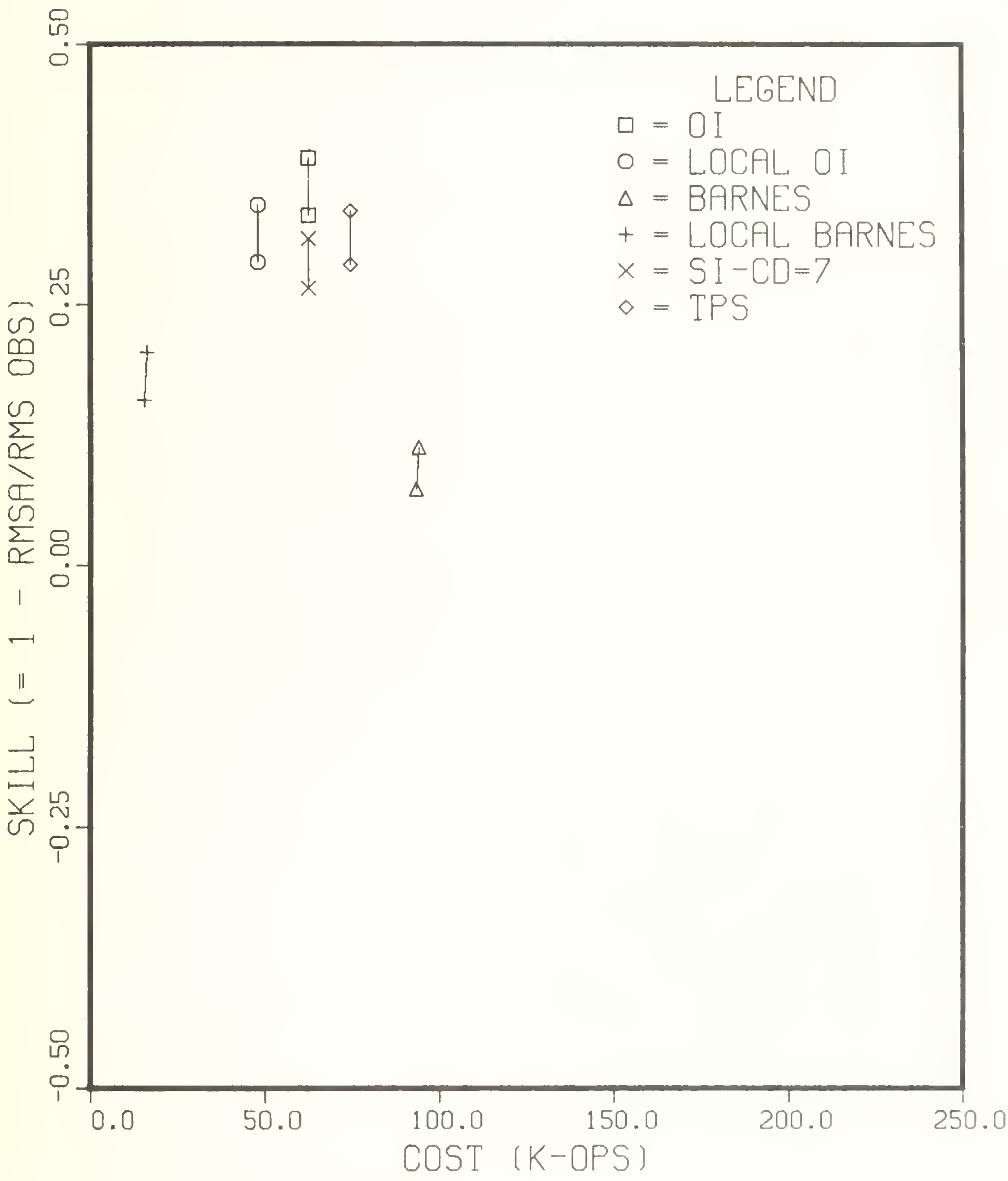
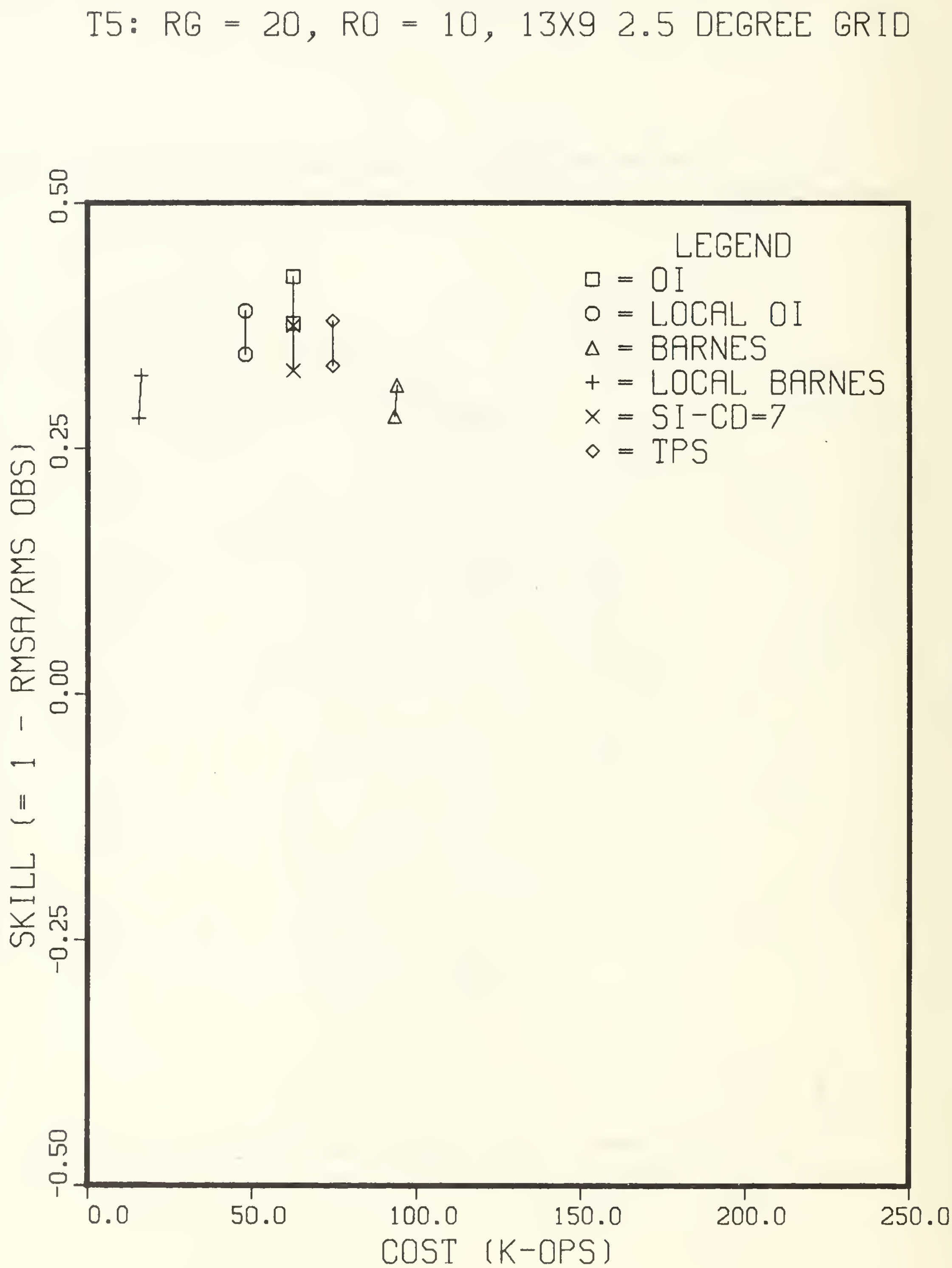


Figure 4



T5: RG = 20, RO = 10, 13X9 2.5 DEGREE GRID
LEGEND
□ = OI
○ = LOCAL OI
△ = BARNES
+ = LOCAL BARNES
× = SI-CD=7
◇ = TPS
SKILL (= 1 - RMSA/RMS OBS)
0.50
0.25
0.00
-0.25
-0.50
0.0
50.0
100.0
150.0
200.0
250.0
COST (K-OPS)

Figure 5

T6: RG = 30, RO = 5, 13X9 2.5 DEGREE GRID

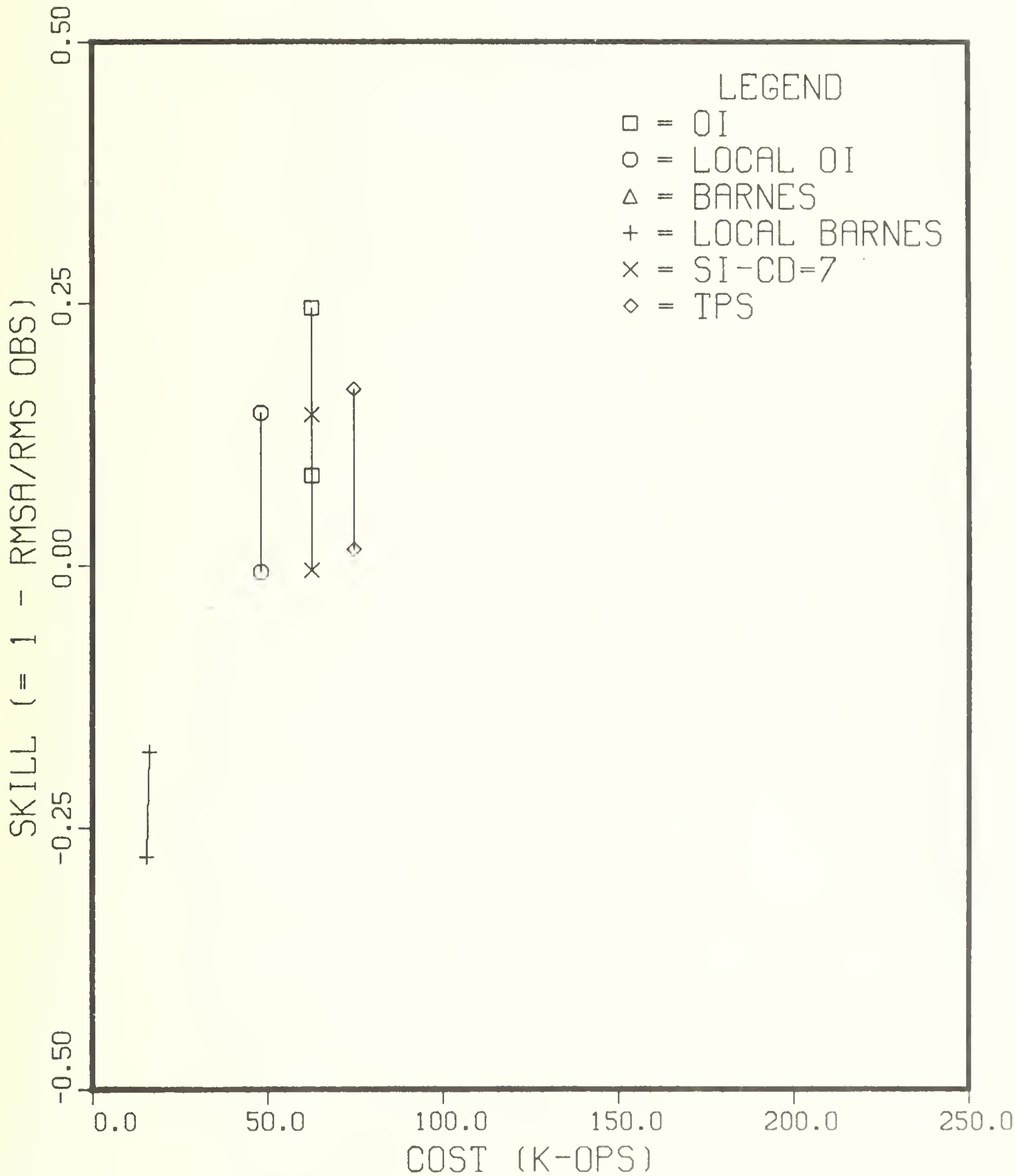


Figure 6

T7: RG = 30, RO = 10, 13X9 2.5 DEGREE GRID

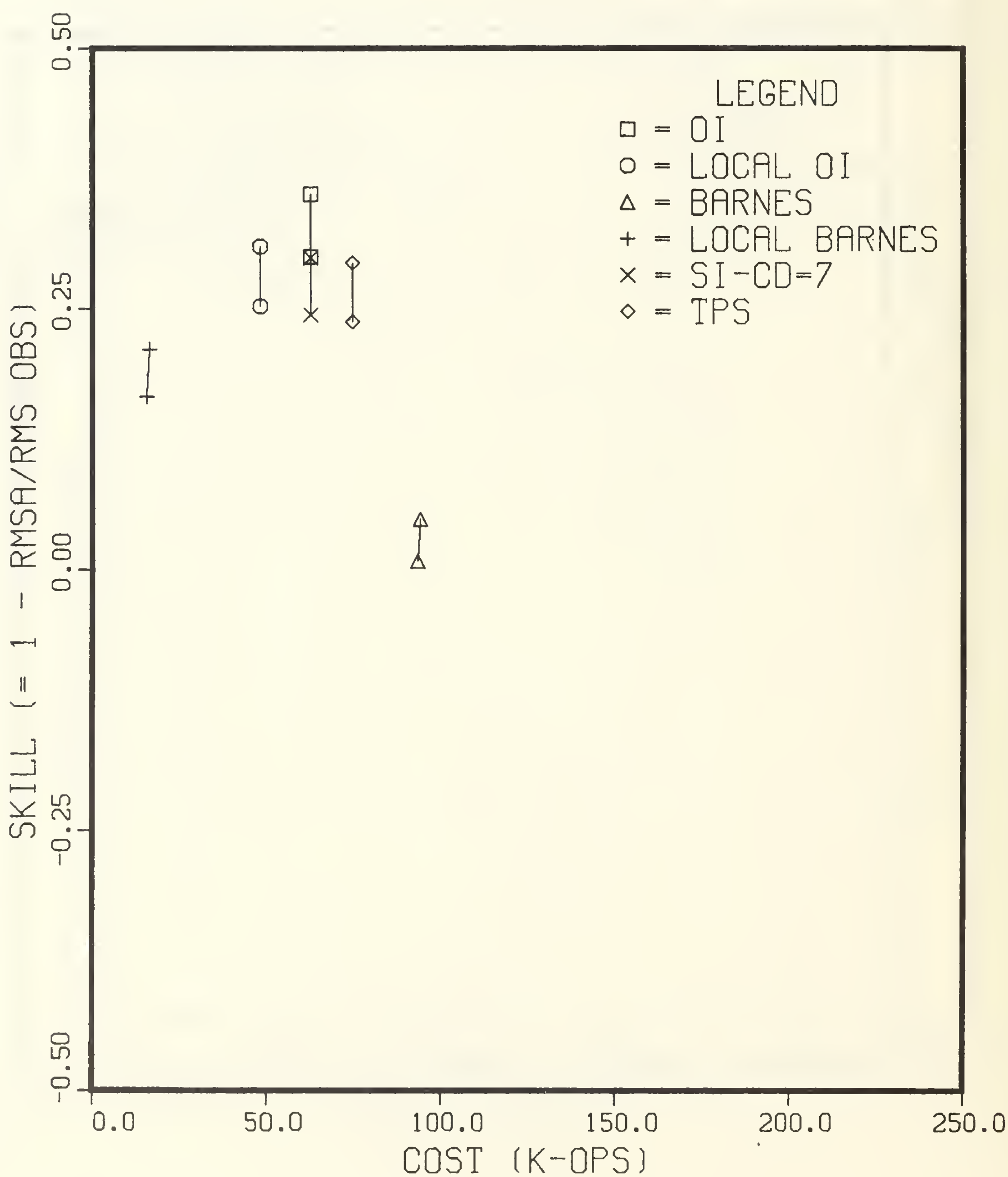


Figure 7

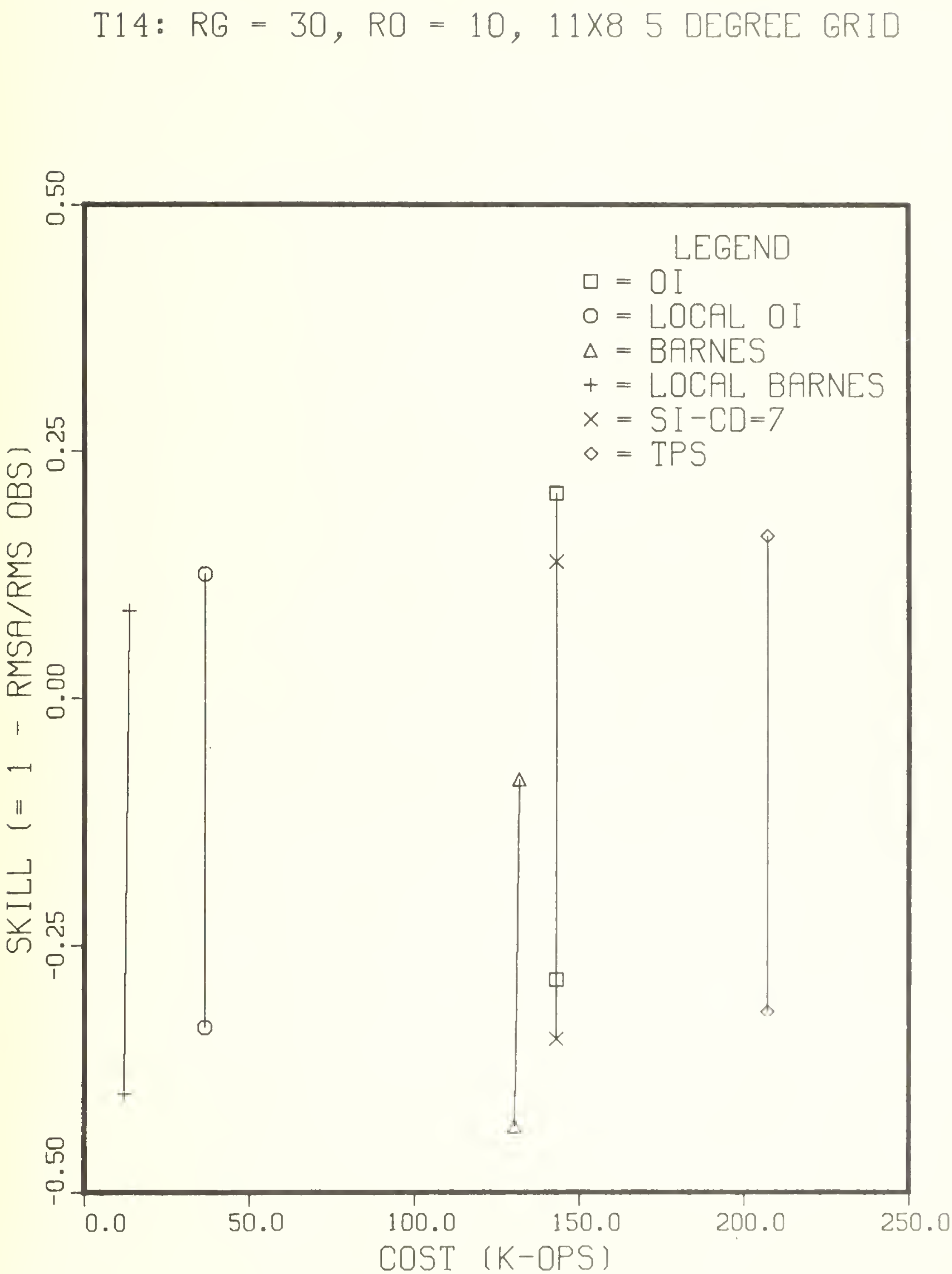

T14: RG = 30, RO = 10, 11X8 5 DEGREE GRID
LEGEND
□ = OI
○ = LOCAL OI
△ = BARNES
+ = LOCAL BARNES
× = SI-CD=7
◇ = TPS
SKILL (= 1 - RMSA/RMS OBS)
0.50
0.25
0.00
-0.25
-0.50
0.0
50.0
100.0
150.0
200.0
250.0
COST (K-OPS)

Figure 8

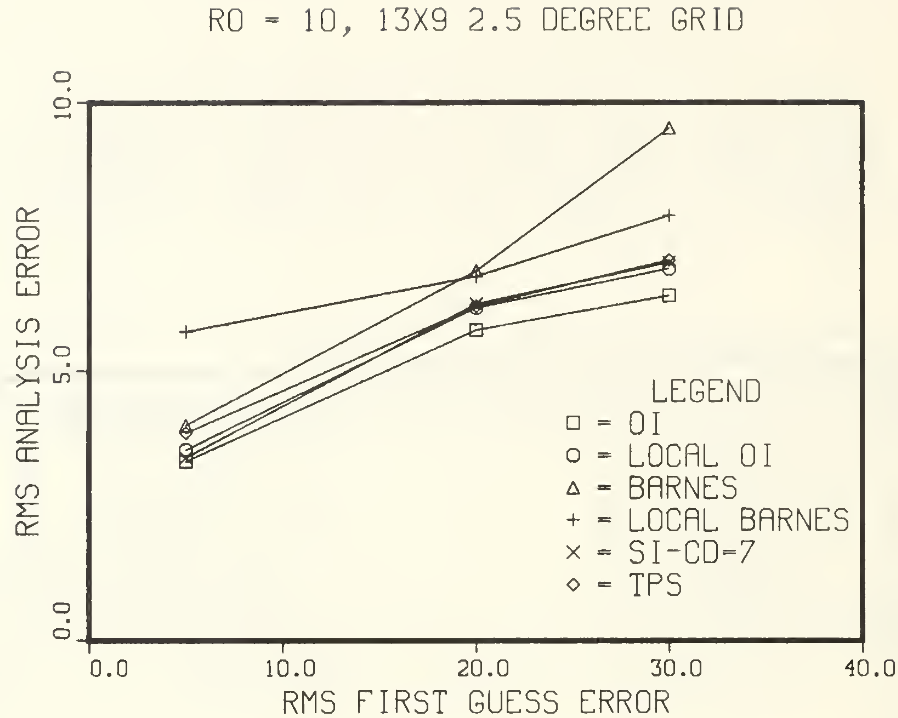


Figure 9

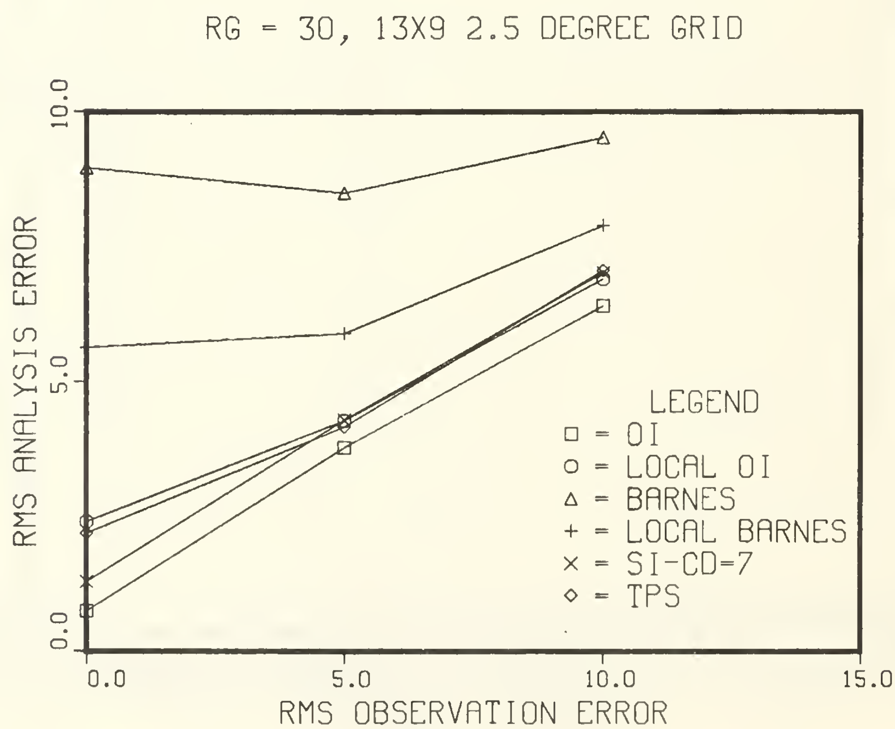


Figure 10

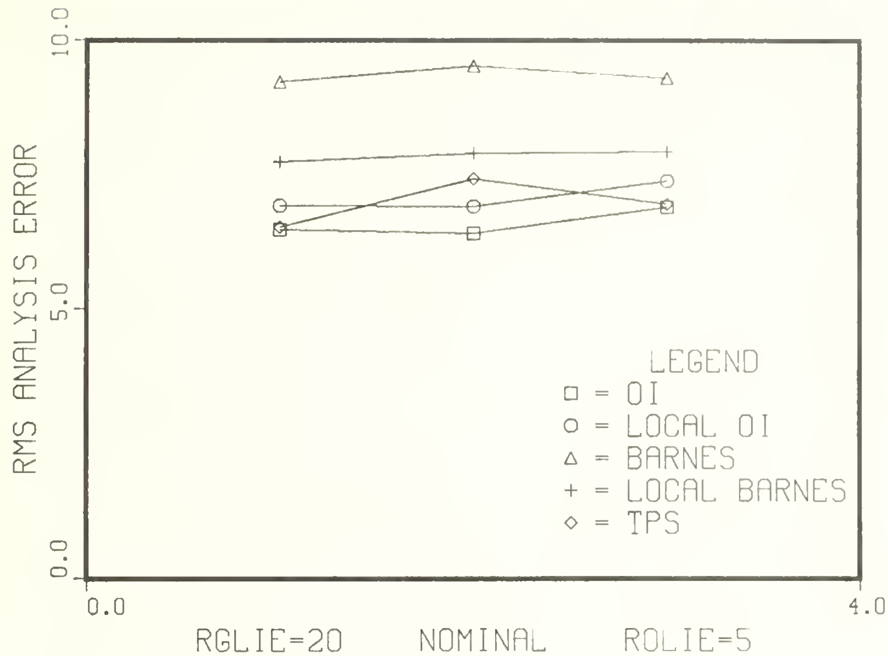

10.0
5.0
0.0
RMS ANALYSIS ERROR
LEGEND
□ = OI
○ = LOCAL OI
△ = BARNES
+ = LOCAL BARNES
◇ = TPS
0.0
4.0
RGLIE=20
NOMINAL
ROLIE=5

Figure 11

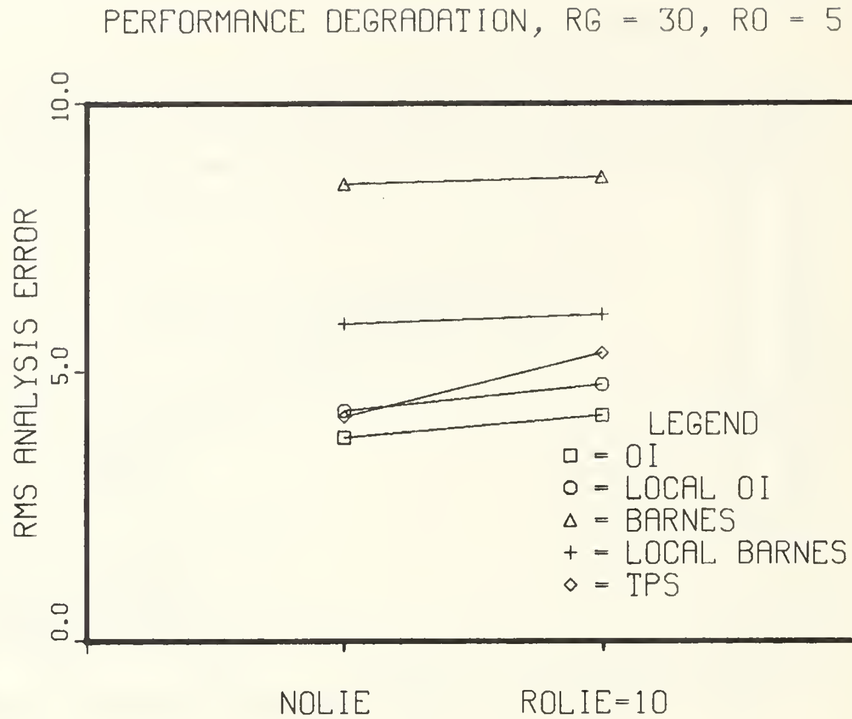


Figure 12

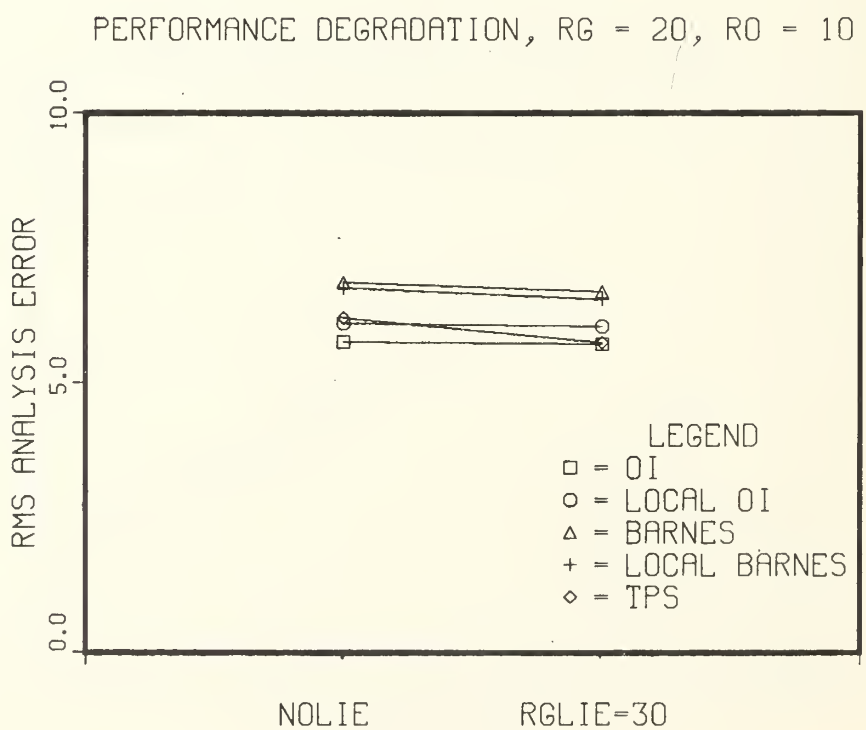


Figure 13

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